Title: Sheaves And Supplying Homological Algebra

Semester: Spring, 2018

Instructor: Alexey Gorodentsev

Course description: The theory of sheaves is a commonly used tool for handling locally defined objects on a manifold, e.g., functions with a restricted domain of definition, local sections of vector bundles, locally defined continuous mappings, etc. In the algebraic/differential geometry and topology it allows to produce global geometric/topological invariants of the manifold from those local data. In the non-commutative geometry it gives various geometric style invariants for categories equipped with Grothendieck's topology. We introduce the basic notions of the sheaf theory, discuss in details the underlying homological algebra (including the cohomology of sheaves), and give some applications of this technique, such as the De Rham theorem comparing the singular and De Rham comohomologies of a smooth manifold.

Prerequisites: The first 3 semesters (6 modules) of the standard courses in Algebra, Calculus, and Geometry/Topology. Some experience in algebraic geometry/topology (projective spaces, topological spaces, simplicial complexes, (co) homology groups) is desirable but not essential.

Curriculum:

- Categories, functors, presheaves. Working examples: the open sets of a topology and the (semi) simplicial sets. Categories of functors, the Yoneda lemma. Adjoint functors. (Co) limits of diagrams. ([GM], [W])

- Sheaves on topological spaces. Stalks and the étalé space of a sheaf. The sheafification. Pull backs and push forwards. Abelian sheaves. ([I], [GM])

- Complexes and (co)homologies, the long exact sequence of homologies. The Koszul complexes. Cohomologies commute with filtered colimits. Spectral sequences for filtered complexes, bicomplexes, and exact couples. ([GM], [D], [W])

- Global sections, flabby sheaves, and the Godement resolution. The sheaf cohomology and hypercohomology. Acyclic resolutions. The Mayer–Vietoris exact sequence and Čech resolution. Acyclic coverings and the Cartan criterion for acyclicity. The Čech cohomology. ([I], [D])

- Fine and soft sheaves. The sheaves of differetial forms, the Poincaré lemma and De Rham theorem. ([GM], [GH], [D])

- Higher direct images. The Leray spectral sequence. ([I], [D], [GH])
- Coherent sheaves in algebraic geometry: examples and applications. Acyclicity of affine varieties. Cohomologies of invertible sheaves on projective spaces. ([D])
- If the time allows: the Grothendieck topologies and sheaves on sites. ([GM])

<u>Textbooks</u>:

[D] V. I. Danilov, Cohomology of Algebraic Manifolds. In: Cohomology of Algebraic Varieties. Algebraic Surfaces, Springer's Encyclopaedia of Mathematical Sciences, Book 35 (Algebraic Geometry 2).

[GM] S. I. Gelfand and Yu. I. Manin, Methods of Homological Algebra, Part I.

[GH] P. Griffiths and J. Harris, Principles of Algebraic Geometry, Vol. I.

[I] B. Iversen, Cohomology of Sheaves.

[W] C. A. Weibel, An Introduction to Homological Algebra.