Lines and conics on the projective plane

AG$2\frac{1}{2}$◇1 (Dezargus theorem). For two triangles $\triangle a_1b_1c_1$, $\triangle a_2b_2c_2$ in $\mathbb{P}_2$, show that the points $(a_1b_1)\cap(a_2b_2)$, $(b_1c_1)\cap(b_2c_2)$, $(c_1a_1)\cap(c_2a_2)$ are collinear if and only if the lines (a_1a_2), (b_1b_2), (c_1c_2) are concurrent. (Such triangles are called *perspective*.)

AG$2\frac{1}{2}$◇2. Show that two triangles are perspective if and only if they are polar one to the other with respect to some smooth conic.

AG$2\frac{1}{2}$◇3. Show that the associated triangle of a quadrangle inscribed in a smooth conic is self-polar with respect to this conic.

AG$2\frac{1}{2}$◇4. For triangles $\triangle a_1b_1c_1$, $\triangle a_2b_2c_2$ inscribed in the same smooth conic, show that the triangle with vertexes $(a_1b_1)\cap(a_2b_2)$, $(b_1c_1)\cap(b_2c_2)$, $(c_1a_1)\cap(c_2a_2)$ and the triangle with sides (a_1a_2), (b_1b_2), (c_1c_2) are perspective.

AG$2\frac{1}{2}$◇5. For two different involutions $\sigma_1,\sigma_2 : C \xrightarrow{\sim} C$ on a smooth conic C,

a) find the number of points $p\in C$ such that $\sigma_1(p)=\sigma_2(p)$

b) show that $\sigma_1\sigma_2=\sigma_2\sigma_1$ if and only if the fixed points of σ_1 are harmonic to the fixed points of σ_2

AG$2\frac{1}{2}$◇6. Let a simple pencil of conics L have the base points p_1, p_2, p_3, p_4 and the singular conics S_1, S_2, S_3. For a smooth conic $C\in L$ compare the cross-ratios $[p_1,p_2,p_3,p_4]$ on C and $[S_1,S_2,S_3,C]$ on L.

AG$2\frac{1}{2}$◇7. Drawn on a sheet of paper are two lines intersecting in some point p outside the sheet. By means of the ruler, plot the line passing trough p and a given point on the sheet.

AG$2\frac{1}{2}$◇8. Marked on a wall are two points quite far from one other. Draw the line joining them by means of a «compound ruler» shorter than the distance between the points.

AG$2\frac{1}{2}$◇9. By means of the ruler, draw the tangent line to a given smooth conic at a given point of the conic.

AG$2\frac{1}{2}$◇10. By means of the ruler, draw a triangle whose vertexes lie on a given smooth conic C and (extensions of) sides pass through given points $a, b, c \notin C$. How many solutions may have this problem?

HINT. For $p\in C$, let $\gamma(p)\in C$ be the return point after the naive attempt to draw such a triangle starting from p and successively passing through a, b, c with two intermediate vertexes on C. Is the map $\gamma : C\to C$, $p\mapsto\gamma(p)$, a homography?

AG$2\frac{1}{2}$◇11. Formulate and solve the problem projectively dual to the previous one.

Individual report card of ____ .
(write your name and surname)

Task $2\frac{1}{2}$ (optional)

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